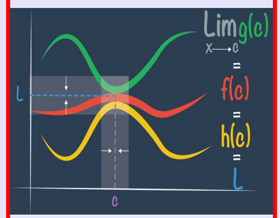
Math 261

Spring 2023

Lecture 48

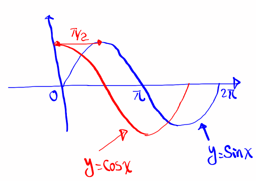


Feb 19-8:47 AM

Suppose $f(x)$ is cont., Prove

$$\int_0^{\pi/2} f(\cos x)\,dx = \int_0^{\pi/2} f(\sin x)\,dx$$

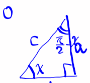
Bad Subs.
$$\begin{cases} u = \sin x & du = \cos x\,dx \\ x = 0 \quad u = 0 & \Rightarrow \int_0^{\pi/2} f(\sin x)\,dx = \int_0^1 f(u) \cdot\, ? \\ x = \pi/2 \quad u = 1 & \end{cases}$$



$u = \frac{\pi}{2} - x \qquad du = -\,dx \rightarrow -du = dx$

$x = 0 \rightarrow u = \frac{\pi}{2}$

$x = \frac{\pi}{2} \rightarrow u = 0$



$\sin u = \sin\left(\frac{\pi}{2} - x\right)$

$= \cos x$

$\cos x = \frac{b}{c} \qquad \sin\left(\frac{\pi}{2} - x\right) = \frac{b}{c}$

$$\boxed{\int_0^{\pi/2} f(\cos x)\,dx} = \int f(\sin u) \cdot -du$$

$$= -\int_{\pi/2}^{0} f(\sin u)\,du$$

Look this up

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$= -\cdot - \int_0^{\pi/2} f(\sin u)\,du$$

$$= \boxed{\int_0^{\pi/2} f(\sin x)\,dx}$$

May 10-9:42 AM

Evaluate $\int_1^{18} \sqrt{\frac{3}{x}}\, dx = \int_1^{18} \frac{\sqrt{3}}{\sqrt{x}}\, dx = \int_1^{18} \sqrt{3}\cdot x^{-1/2}\, dx$

$$= \sqrt{3} \cdot \frac{x^{-1/2+1}}{-1/2+1}\Bigg]_1^{18}$$

$$= \sqrt{3} \cdot \frac{x^{1/2}}{1/2}\Bigg]_1^{18} = 2\sqrt{3}\cdot\sqrt{x}\Bigg]_1^{18}$$

$$= 2\sqrt{3}\left[\sqrt{18} - \sqrt{1}\right] = 2\sqrt{3}\left[\sqrt{9}\sqrt{2} - 1\right]$$

$$= \boxed{2\sqrt{3}\left(3\sqrt{2} - 1\right)}$$

May 11-8:51 AM

Evaluate

$$\int_1^{64} \frac{1 + \sqrt[3]{x}}{\sqrt{x}}\, dx = \int_1^{64} \left(\frac{1}{\sqrt{x}} + \frac{\sqrt[3]{x}}{\sqrt{x}}\right) dx$$

$$= \int_1^{64} \left(x^{-1/2} + x^{\frac{1}{3} - \frac{1}{2}}\right) dx = \int_1^{64} \left(x^{-1/2} + x^{-1/6}\right) dx$$

$$= \left(\frac{x^{-1/2+1}}{-1/2+1} + \frac{x^{-1/6+1}}{-1/6+1}\right)\Bigg]_1^{64} = \left(\frac{x^{1/2}}{1/2} + \frac{x^{5/6}}{5/6}\right)\Bigg]_1^{64}$$

$$= \left[2\sqrt{x} + \frac{6}{5}\sqrt[6]{x^5}\right]\Bigg]_1^{64}$$

$$= \left(2\sqrt{64} + \frac{6}{5}\sqrt[6]{64^5}\right) - \left(2\sqrt{1} + \frac{6}{5}\sqrt[6]{1^5}\right)$$

$$= 2\cdot 8 + \boxed{\frac{6}{5}\cdot 32} - 2\ \boxed{-\frac{6}{5}}$$

$$= 14 + \frac{6}{5}(32 - 1) = 14 + \frac{6\cdot 31}{5} = \boxed{\frac{256}{5}}$$

May 11-8:55 AM

Find $\int \frac{1+\sqrt[3]{x}}{\sqrt{x}}\,dx$

Let $x = u^6 \rightarrow u = \sqrt[6]{x}$

$dx = 6u^5\,du$

$$\int \frac{1+\sqrt[3]{x}}{\sqrt{x}}\,dx = \int \frac{1+\sqrt[3]{u^6}}{\sqrt{u^6}} \cdot 6u^5\,du$$

$$= \int \frac{1+u^2}{\cancel{u^3}} \cdot 6\cancel{u^5}^{\,u^2}\,du = 6\int (1+u^2)\cdot u^2\,du$$

$$= 6\int (u^2+u^4)\,du = 6\left(\frac{u^3}{3}+\frac{u^5}{5}\right) + C$$

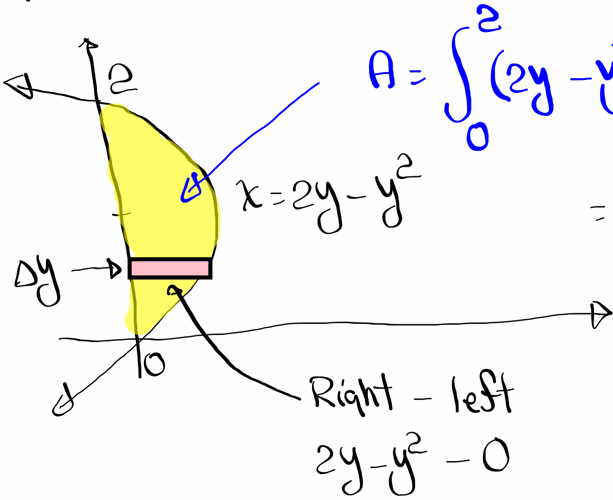
$$= 2u^3 + \frac{6}{5}u^5 + C$$

$$= 2\left(\sqrt[6]{x}\right)^3 + \frac{6}{5}\left(\sqrt[6]{x}\right)^5 + C$$

$$= \boxed{2\sqrt{x} + \frac{6}{5}\sqrt[6]{x^5} + C}$$

May 11-9:03 AM

Find the shaded area below:



$$A = \int_0^2 (2y - y^2)\cdot dy$$

$$= \left(y^2 - \frac{y^3}{3}\right)\Big]_0^2$$

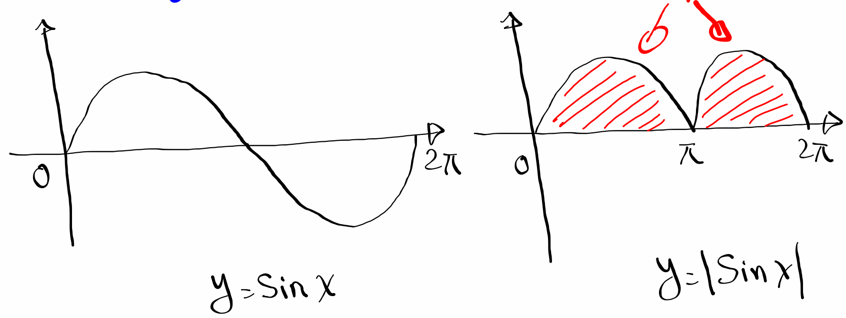
$$= 2^2 - \frac{2^3}{3} - 0$$

$$= 4 - \frac{8}{3} = \boxed{\frac{4}{3}}$$

May 11-9:10 AM

Evaluate $\int_0^{2\pi} |\sin x|\, dx = 2\int_0^{\pi} \sin x \, dx$

Equal Areas



$= 2 \cdot -\cos x \Big]_0^{\pi} = -2\left[\cos\pi - \cos 0\right]$

$= -2\left[-1 - 1\right] = \boxed{4}$

May 11-9:15 AM

Evaluate $\int_0^4 \frac{x}{\sqrt{1+2x}}\, dx$

$u = 1 + 2x$

$\frac{u-1}{2} = x$

$du = 2dx$

$\frac{du}{2} = dx$

$x = 0 \to u = 1$

$x = 4 \to u = 9$

$\int_1^9 \frac{\frac{u-1}{2}}{\sqrt{u}} \cdot \frac{du}{2}$

$= \int_1^9 \frac{\frac{1}{2}(u-1)}{2\sqrt{u}}\, du$

$= \frac{1}{4}\int_1^9 \frac{u-1}{\sqrt{u}}\, du = \text{. . . . . .} = \boxed{}$

May 11-9:19 AM

If $f(x)$ is cont. on $[a,b]$, then average value of $f(x)$ on $[a,b]$ is

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Class QZ 13:

Find the average value of $f(x)=\sqrt[3]{x}$ on $[1,8]$. Exact answer required.

$$f_{ave} = \frac{1}{8-1}\int_1^8 \sqrt[3]{x}\,dx = \frac{1}{7}\int_1^8 x^{1/3}\,dx = \frac{1}{7}\cdot\frac{x^{4/3}}{4/3}\Big]_1^8$$

$$= \frac{3}{28}\left(8^{4/3} - 1^{4/3}\right) = \frac{3}{28}(16-1) = \frac{3}{28}\cdot 15 = \boxed{\frac{45}{28}}$$

May 11-9:24 AM